CORE MATHS – PREPARATORY WORK

Before starting an Advanced Level course you must ensure you can do the basics of algebra quickly and accurately.
This sheet contains a set of worked examples and questions to help you make sure you are ready for A level. It is written to highlight the key methods in algebra and to help you avoid the errors made by previous A Level students. Study each method carefully before trying to apply it and make sure you understand what you are trying to do.

SE 2008

Section A: Expanding Brackets

The key to this section is knowing how to add, subtract and multiply negative numbers.

A1 $3(x^2 - y) - 2x(y - 4x)$ — Multiply out each bracket

$= 3x^2 - 3y - 2xy + 8x^2$ — Note that you are multiplying the second bracket by -2x not 2x and that: - 2x × -4x = + 8x²

$= 11x^2 - 3y - 2xy$ — Collect the x^2 terms. No further simplification is possible.

A2 $(2x + 3y)(5x - 2y)$ — Multiply out to achieve 4 terms
Note that – x + = –

$= 10x^2 - 4xy + 15xy - 6y^2$ — Collect the term in xy carefully (noting that -4 + 15 = 11)

$= 10x^2 + 11xy - 6y^2$ — No further simplification is possible.

A3 $(x + 3y)^2$ — $(x + 3y)^2$ is **not** $x^2 + 3y^2$ nor is it $x^2 + 9y^2$!!

$= (x + 3y)(x + 3y)$ — Squaring means multiply by itself.

$= x^2 + 3xy + 3xy + 9y^2$ — Multiply out to achieve 4 terms

$= x^2 + 6xy + 9y^2$ — Collect the xy terms to simplify.

SECTION A QUESTIONS

Expand and simplify

1) 3(x – 2y) – 2x(y – 4)
2) x(x – 3) + 2(x – 3)
3) (x – 5)(x + 3)
4) (2x – 1)(3x + 2)
5) (3x – 5)(3x + 5)
6) $(x – 5y)^2$
7) 5 – 3(x – 2) (hint BIDMAS)
8) $2(x + 1)^2$ (hint BIDMAS)
9) (2x – 3y)(4x – 5y)
10) $(8 – x)^2$

Section B: Factorising Expressions

Factorising into 1 bracket (common factors)

B1 $12x^3y - 18x^2y^3$

- State the question. Look to factorise completely.

$= 6x^2y\ (\qquad)$

- The highest number that each term divides by is 6.
- The highest power of x in each term is x^2.
- The highest power of y in each term is y.
- These are the common factors to be pulled outside of the bracket.

$= 6x^2y\ (2x - 3y^2)$

- Now divide each term by the common factors to get the contents of the bracket.
- The terms in the bracket have no common factors so you've factorised completely.
- Check by expanding that you've factorised correctly.

Factorising Quadratics (Intermediate Level)

Standard 3 term quadratic…

B2 $x^2 + x - 12$

- It's a 3 term quadratic and it only has x^2 so if it will factorise it will go into 2 brackets $(x\quad)(x\quad)$.

$(x + \ )(x - \ )$

- The –12 at the end means the numbers in the bracket must multiply to make -12. To get a negative by multiplying the signs must be different.

$(x + 4)(x - 3)$

- You need to try different combinations (-12 & 1, 12 & -1, -6 & 2, 6 & -2, -4 & 3, 4 & -3) and mentally multiply out until you get + 1x as your middle term.
- There is only one possible combination that works.

Only 2 terms – missing constant term…

B3 $x^2 + 5x$

- It's a 2 term quadratic with a missing constant term so if it will factorise into one bracket by common factor.

$x(\ x + 5\ )$

- The only common factor is x. There is only one possible combination that works.

Only 2 terms – missing x term…

B4 $x^2 + 5$

- This expression will not factorise. The only quadratics of this sort that will are the difference of two squares sort – see next example.

B5 $x^2 - 4$

$(x + \quad)(x - \quad)$

$(x + 2)(x - 2)$

- $x^2 - 4$ is a special kind of quadratic called the difference of two squares (dots!) because it is one square number subtract another square number.
- Dots factorises into (√1st term + √2nd term) (√1st term + √2nd term)
- Check by expansion that $(x + 2)(x - 2)$ gives $(x^2 + 0x - 4)$.

Factorising Quadratics (Higher Level)

Look for common factors first…

B6 $2x^2 - 8x + 6$

- It is a standard 3 term quadratic so will factorise into 2 brackets.
- There is a common factor of 2. Pull this out of each term to give an easier quadratic to factorise.

$= 2(x^2 - 4x + 3)$

$= 2(x \quad)(x \quad)$

- The + 3 at the end shows the signs in the brackets must be the same.

$= 2(x - \quad)(x - \quad)$

- The – 4x is negative so the signs in the brackets must both be –.

$= 2(x - 3)(x - 1)$

- This means the only possible values are –3 & -1.
- Check by expansion that $(x - 3)(x - 1)$ gives $(x^2 - 4x + 3)$.

B7 $9x^2 - 81$

- State the question. There is a common factor of 9. Pull this out of each term to give an easier quadratic to factorise.

$= 9(x^2 - 9)$

- $x^2 - 9$ is a difference of two squares (dots!) because it is one square number subtract another square number.

$= 9(x + \quad)(x - \quad)$

$= 9(x + 3)(x - 3)$

- Dots factorises into (√1st term + √2nd term) (√1st term + √2nd term)
- Check by expansion that $(x + 3)(x - 3)$ gives $(x^2 + 0x - 9)$.

Standard 3 term quadratic…

B8 $4x^2 + 8x - 5 = 0$

- State the question. There are no common factors.
- It's a quadratic so if it will factorise it will go into 2 brackets but as it is $4x^2$ then it could be either $(4x \quad)(x \quad)$ or $(2x \quad)(2x \quad)$.

$(\_x + \quad)(\_x - \quad) = 0$

- The –5 at the end means the signs in the bracket are different and the end nos are either 5 & -1 or –5 & 1.
- You need to try different combinations and mentally multiply out until you get + 8x as your middle term.

$(2x + 5)(2x - 1) = 0$

- There is only one possible combination that works.

SECTION B QUESTIONS

Factorise completely

1) $6pq - 15p^2$
2) $12ab^2 - 8a^2b$
3) $x^2 - 36x$
4) $x^2 - 36$
5) $x^2 + 5x + 6$
6) $x^2 - 5x - 6$
7) $2x^2 + 6x - 16$
8) $2x^2 - 14x + 12$
9) $6x^2 - 6$
10) $14x^2 - 3x - 2$

Section C: Solving Quadratics

Solving quadratics by factorising

C1 $4x^2 - 17x - 15 = 0$

- There are no common factors.
- It's a quadratic so if it will factorise it will go into 2 brackets but as it is $4x^2$ then it could be either (4x)(x) or (2x)(2x).

(__x +)(__x –) = 0

- The –15 at the end means the signs in the bracket are different and the end nos are either 15 & -1, -15 & 1, -5 & 3 or 5 & -3.
- You need to try different combinations and mentally multiply out until you get -17x as your middle term.

(4x + 3)(x – 5) = 0

- There is only one possible combination that works.

You must now solve the equation (ie find x). You have two things multiplied together that equal 0. This means that one or both of them must equal 0.

Either (4x + 3) = 0 which means 4x = – 3 so x = **– $^3/_4$**

Or (x – 5) = 0 x = 5

Solving quadratics by the formula

C2 Solve $2x^2 - 3x - 6 = 0$

- **Only use the formula to solve a quadratic if you are unable to factorise into 2 brackets.**
- Always state the formula and the values a= , b= and c= first.
- Substitute carefully before starting to simplify.
- Note that – – 3 = + 3 , $(-3)^2 = +9$ and that the – 4 x 2 x – 6 becomes + 48
- Simplify to give 2 answers. If it says **exact form** leave them as surds.
- If you need to give your answer to 3sf for example be careful how you type it into your calculator. Either press equals after typing 3 + √57 or put brackets around it.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 2, \ldots b = -3, \ldots c = -6$$

$$x = \frac{--3 \pm \sqrt{(-3)^2 - 4 \times 2 \times -6}}{2 \times 2}$$

$$x = \frac{3 \pm \sqrt{(+).9 + 48}}{4}$$

$$x = \frac{3 \pm \sqrt{57}}{4}$$

$$hence \ldots x = \frac{3 + \sqrt{57}}{4} \ldots or \ldots x = \frac{3 - \sqrt{57}}{4}$$

$$ie \ldots x = 2.637458 \ldots or \ldots x = -1.137458 \ldots$$

$$\ldots x = 2.64(3sf) \ldots or \ldots x = -1.14(3sf)$$

SECTION C QUESTIONS

Factorise and then solve these quadratic equations.

1) $x^2 + 13x + 40 = 0$

2) $t^2 - 5t + 6 = 0$

3) $p^2 - 4p = 0$

4) $g^2 - 100 = 0$

5) $4x^2 - 4x - 3 = 0$

6) $6y^2 - 5y - 6 = 0$

7) $8x^2 - 2x - 1 = 0$

Solve using the formula giving your answers to 3sf.

8) $2x^2 + 5x + 1 = 0$

9) $3x^2 - 4x - 2 = 0$

10) $4x^2 - 9x + 3 = 0$

Section D – Rearranging Formulae (I)

Formulae are constructed using the order of operations, BIDMAS…

Brackets	1st
I ndices (powers)	2nd
Division	}3rd
Multiplication	}
Addition	}4th
Subtraction	}

If you rearrange a formula you are 'unpicking it' so you must follow BIDMAS in reverse and do the opposite operation to both sides.

D1 Make p the subject of this formula $y = \frac{4p+3}{7}$

This is the same as y = (4p + 3) ÷ 7 because all of the 4p + 3 is in the numerator of the fraction.
Think about how it was constructed… p (×4) (+3) (÷7) = y.
To make p the subject you must follow these steps backwards and do the opposite.

Solution:

$$y = \frac{4p+3}{7}$$

- To undo the ÷7 you ×7 both sides $7y = 4p + 3$
- To undo the +3 you –3 both sides $7y - 3 = 4p$
- To undo the ×4 you ÷4 both sides $\frac{7y-3}{4} = p......hence....p = \frac{7y-3}{4}$

D2 Make p the subject of this formula $y = \frac{4p}{7} + 3$

Note this appears very similar to example E1 but is not the same because it was constructed in a different order.
This was constructed… p (×4) (÷7) (+3) = y.
To make p the subject you must follow these steps backwards and do the opposite to both sides.

Solution:

$$y = \frac{4p}{7} + 3$$

- To undo the +3 you –3 both sides $y - 3 = \frac{4p}{7}$
- To undo the ÷7 you ×7 both sides $7(y-3) = 4p$
- To undo the ×4 you ÷4 both sides $\frac{7(y-3)}{4} = p$

$$p = \frac{7(y-3)}{4} \quad \text{or} \quad p = \frac{7y-21}{4}$$

D3 Make p the subject of this formula $y = \frac{4p}{7} + 3$

This is exactly the same question as example E2. However there is another method to rearrange it…

You can eliminate the denominator of 7 by **multiplying through** by 7 – that mean multiplying the whole of each side by 7. In effect you multiply each term by 7

Solution:

$$y = \frac{4p}{7} + 3$$

- Multiply **every term** by 7 $(y \times 7) = (\frac{4p}{7} \times 7) + (3 \times 7)$
- Simplify (note: denominator is now gone) $7y = 4p + 21$
- To undo the ×4 you ÷4 both sides $7y - 21 = 4p......hence....p = \frac{7y-21}{4} or...p = \frac{7(y-3)}{4}$

Both methods (from E2 and E3) are equally good. The method of multiplying through is very useful but you must remember to multiply every term.

D4 Make g the subject of this formula $m = f - \frac{6g}{5}$

- The problem with this formula is that the subject that you want, g,is negative. $m + \frac{6g}{5} = f - \frac{6g}{5} + \frac{6g}{5}$
- The easiest way to solve this question without making any errors with minus signs is to add the whole of the term involving g to both sides.
  This now gives a formula with a positive term in g. $m + \frac{6g}{5} = f$

  $\frac{6g}{5} = f - m$
- Now follow the previous method - look at how the formula was made…g (×6) (÷5) (+m) = f $6g = 5(f - m)$
- Reverse the order of the steps and do the opposite to both sides of the formula. $g = \frac{5(f-m)}{6}$

  (Note: the order of construction as g (÷5) (×6) (+m) = f would also be correct)

SECTION D QUESTIONS

Make the letter in brackets the subject of the formula.

1. p = aq – r (q)
2. p = a(q – r) (q)
3. v = u + at (u)
4. v = u + at (a)
5. p(q + r) = 2 (q)
6. $y = \frac{2p}{3} + r$ (p)
7. $y = \frac{2p + r}{3}$ (p)
8. r = 2 – pq (q)
9. $y = r - \frac{qp}{x}$ (p)
10. $a^2 = b^2 + c^2$ (b)

Section E – Rearranging Formulae (II)

E1 Make g the subject of this formula $4f + h = \frac{3}{g}$

- The problem with this formula is that the subject that you want, g, is in the denominator of a fraction.
- The way to solve this problem is to multiply both sides by g. You must put brackets around the 4f + h to indicate that both terms are multiplied by g.This now gives a formula with g no longer in the denominator.
- Now you have g multiplied by a bracket equals 3 so divide both sides by the content of the bracket to get g =…

$$4f + h = \frac{3}{g}$$
$$g \times (4f + h) = \frac{3}{g} \times g$$
$$g(4f + h) = 3$$
$$g = \frac{3}{(4f + h)}$$
$$hence....g = \frac{3}{4f + h}$$

E2 Make x the subject of this formula $y - mx = cx + 4$

- The problem with this formula is that the subject you want, x, appears twice in the formula.
- The way to solve this problem is to collect all the terms With x in on one side and all the terms without x in them on the other side.
- To do that add mx to both sides and –4 to both sides
- You can now pull x out as a common factor by factorising
- Now you have x multiplied by a bracket equals y - 4 so Divide both sides by the content of the bracket to get x =…

$$y - mx = cx + 4$$
$$y = cx + 4 + mx$$
$$y - 4 = cx + mx$$

$$y - 4 = x(c + m)$$
$$\frac{(y - 4)}{(c + m)} = x$$
$$hence...$$
$$x = \frac{y - 4}{c + m}$$

SECTION E QUESTIONS

Make the letter in brackets the subject of the formula.

1. $y = \frac{2p}{q} + r$ (p)
2. $y = \frac{2p}{q} + r$ (q)
3. ax + b = cx + d (x)
4. 2(q + 3) = a(q – r) (q)
5. $p = \frac{5t - r}{u}$ (u)
6. $p = \frac{5t - u}{u}$ (u)
7. p(q + r) = 2(q – p) (q)
8. $p + 3 = \frac{qp + r}{2}$ (p)
9. p(r – q)= 2 + 3(q – r) (r)
10. $y = r - \frac{qp}{x}$ (x)