Quadratic Equations (C1 and C2)

1. Factorising

1.1 Introduction

Factorising quadratic expressions is an essential skill at A-level. In addition to solving quadratic equations, you will need it in many other areas of maths including:

- Solving simultaneous equations (C1)
- Solving trigonometric and exponential equations (C2)

1.2 The technique

This method of factorising may be new to you, but it's worth persevering with as it takes out any guess work, and will ultimately save you a lot of time.

e.g. 1: $6x^2 + 11x + 3$

1: multiply the 1st and 3rd coefficients (6 x 3 = 18).
We are looking for 2 numbers that *multiply* to make **18** and *add* to make the 2nd coefficient, which is **11**. This can be summarized as follows:

$6x^2 + 11x + 3$

x ⟶ 18
\+ ⟶ 11

The 2 numbers are 2 and 9

2: re-write the original equation, but instead of writing 11x, write 2x + 9x. (from above)

So: $6x^2 + 11x + 3 \;=\; 6x^2 + 2x + 9x + 3$

3: Now factorise by group:

$2x(3x+1) + 3(3x+1)$
$= (2x+3)(3x+1)$

Let's look again at stage 2. Would it matter if you wrote 9x + 2x? Let's have a look:
$6x^2 + 9x + 2x + 3$
$= 3x(2x+3) + 1(2x+3)$ *
$= (3x+1)(2x+3)$ - so it still works

* This line looks strange but you **must** factorise 2x + 3. The only factor of 2x **and** 3 is 1.

e.g. 2:

1: $3x^2 + 7x + 2$

x → 6
\+ → 7

The 2 numbers are 6 and 1

2: re-writing: $3x^2 + 6x + x + 2$

3: factorise by group:

$$3x(x+2) + 1(x+2)$$
$$= (3x+1)(x+2)$$

e.g. 3:

1: $6x^2 + 19x + 10$

x → 60
\+ → 19

The 2 numbers are 15 and 4

2: re-writing: $6x^2 + 15x + 4x + 10$

3: factorise by group:

$$3x(2x+5) + 2(2x+5)$$
$$= (3x+2)(2x+5)$$

e.g. 4: This method works equally well if negative numbers are involved:

1: $3x^2 + 7x - 6$

x → -18
\+ → 7

The 2 numbers are 9 and -2

2: re-writing $3x^2 + 9x - 2x - 6$

3: factorise by group:

$$3x(x+3) - 2(x+3)$$
$$= (3x-2)(x+3)$$

1.3 Questions

I. $2x^2 + 5x + 3$
II. $2x^2 + 7x + 3$
III. $3x^2 + 7x + 2$
IV. $2x^2 + 11x + 12$
V. $3x^2 - 5x - 2$
VI. $2x^2 - x - 15$
VII. $2x^2 + x - 21$
VIII. $3x^2 - 5x - 2$
IX. $8x^2 - 10x + 3$
X. $12x^2 + 23x + 10$
XI. $4x^2 - 23x + 15$
XII. $6x^2 - 27x + 30$

2. Solving Quadratic Equations by Factorising

2.1 Introduction

Often, the easiest way to solve quadratic equations is to factorise it. Even at C4, after 10 lines of complex maths you will often be left with a quadratic equation that factorises.

2.2 The technique

Let's look at the original example again, only this time makes it equal to zero.

$6x^2 + 11x + 3 = 0$

As we have seen, this factorises to:

$(2x + 3)(3x + 1) = 0$

By putting each bracket equal to zero,

We have: $2x + 3 = 0$ **OR** $3x + 1 = 0$

$x = -\frac{3}{2}$ or $x = -\frac{1}{3}$

2.3 Questions

I. $2x^2 - 3x - 2 = 0$
II. $3x^2 + 10x - 8 = 0$
III. $2x^2 + 7x - 15 = 0$
IV. $6x^2 - 13x + 6 = 0$
V. $4x^2 - 29x + 7 = 0$
VI. $10x^2 - x - 3 = 0$
VII. $12x^2 - 16x + 5 = 0$
VIII. $6x^2 + x - 1 = 0$
IX. $4x^2 - 3x - 10 = 0$
X. $6x^2 + 17x - 3 = 0$

3. Using the formula

3.1 Introduction

Although C1 is non-calculator, this method of solving quadratic equations is still useful when dealing with surds and deciding if a quadratic equation has real roots. You will also be expected to solve quadratic equations using the formula in C2 and M1.

3.2 The technique

For any quadratic equation in the form $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is fairly straight forward substitution if a, b and c are all positive.

You need to be more careful with the substitutions if b or c are negative.

e.g. 1 $3x^2 - 4x - 5 = 0$

a = 3 b = -4 c = -5

Substituting: $x = \frac{4 \pm \sqrt{(-4)^2 - 4(3)(-5)}}{2(3)}$ [note how the 4 at the start is NOT a -4]

$$x = \frac{4 \pm \sqrt{16 - (-60)}}{6}$$ [This line is important to avoid silly mistakes]

$$x = \frac{4 \pm \sqrt{76}}{6}$$ [16 – (-60) = 16 + 60]

So x = 2.12 or -0.79 to 2 d.p.

3.3 Questions

I. $2x^2 + 6x + 3 = 0$
II. $x^2 + 4x + 1 = 0$
III. $5x^2 - 5x + 1 = 0$
IV. $x^2 - 7x + 2 = 0 = 0$
V. $2x^2 + 5x - 1 = 0$
VI. $3x^2 + x - 3 = 0$
VII. $3x^2 + 8x - 6 = 0$
VIII. $3x^2 - 7x - 20 = 0$
IX. $2x^2 - 7x - 15 = 0$
X. $x^2 - 3x - 2 = 0$

4. Completing the Square

4.1 Introduction

You may not have been taught this last year, but it is another extremely useful technique for finding exact solutions of quadratic solutions without having to use the formula. It can also be used to find maximum and minimum values on a curve.

4.2 The Technique

Look at the following expansions:

$$(x+1)^2 = (x+1)(x+1) = x^2 + 2x + 1$$
$$(x+2)^2 = (x+2)(x+2) = x^2 + 4x + 4$$
$$(x+3)^2 = (x+3)(x+3) = x^2 + 6x + 9$$

You should be able to spot a pattern. Could you expand $(x+11)^2$? [$x^2 + 22x + 121$]

When you expand $(x+a)^2$ you get $x^2 + 2ax + a^2$

So, if we wanted to complete the square for $x^2 + 4x$, we know it must have come from the expansion of $(x+2)^2$, because 2 is ½ of 4. However, when you expand $(x+2)^2$, you get $x^2 + 4x + 4$

Because we don't need the +4 we can say $x^2 + 4x = (x+2)^2 - 4$

e.g.1 $x^2 + 8x = (x+4)^2 - 4^2 = (x+4)^2 - 16$
e.g.2 $x^2 + 10x = (x+5)^2 - 5^2 = (x+5)^2 - 25$
e.g.3 $x^2 + x = (x+0.5)^2 - 0.5^2 = (x+0.5)^2 - 0.25$
e.g.4 $x^2 - 6x = (x-3x)^2 - 3^2 = (x-3)^2 - 9$

For something like $x^2 + 8x$ -3, just apply this technique to the $x^2 + 8x$, but then subtract 3.

e.g.5 $x^2 + 8x - 3 = (x+4)^2 - 4^2 - 3 = (x+4)^2 - 19$
e.g.6 $x^2 + 10x + 4 = (x+5)^2 - 5^2 + 4 = (x+5)^2 - 21$
e.g.7 $x^2 + x - 1.25 = (x+0.5)^2 - 0.5^2 - 1.25 = (x+0.5)^2 - 1.5$
e.g.8 $x^2 - 6x + 4 = (x-3x)^2 - 3^2 + 4 = (x-3)^2 - 5$

4.3 Questions

Complete the square for the following expressions:

I. $x^2 + 6x + 13$
II. $x^2 - 8x + 5$
III. $x^2 + 10x + 15$
IV. $x^2 - 2x + 7$
V. $x^2 + 12x + 50$
VI. $x^2 - 4x + 6$
VII. $x^2 + x + 2.75$
VIII. $x^2 - 20x + 99$
IX. $x^2 - x - 5.75$
X. $x^2 + \frac{2}{3}x - \frac{2}{9}$

4.4 Solving equations

The technique of completing the square can be used to solve quadratic equations that cannot be factorised.

e.g. 1 $x^2 + 8x - 3 = 0$

1. *Complete the square*:

$(x+4)^2 - 4^2 - 3 = 0$

$(x+4)^2 - 19 = 0$

2. *Rearrange to make x the subject:*

$(x+4)^2 = 19$

$(x+4) = \sqrt{19}$

$x = -4 \pm \sqrt{19}$ [Don't forget there are 2 answers because√ 19 can be + or -]

4.5 Questions

I. $x^2 - 8x + 12 = 0$
II. $x^2 + 10x + 21 = 0$
III. $x^2 - 4x - 5 = 0$
IV. $x^2 + 4x - 3$
V. $x^2 + 12x - 1 = 0$
VI. $x^2 + 6x - 4 = 0$
VII. $x^2 + 12x + 34 = 0$
VIII. $x^2 - 10x + 15 = 0$
IX. $x^2 - 3x - \frac{3}{4} = 0$
X. $x^2 - x - 1.75 = 0$

4.6 Finding the Minimum Points of an x^2 Graph

A second important use for completing the square is to find the minimum point of an x^2 graph.

e.g. 1 $y = x^2 + 8x - 3$

1. *Complete the square*:

y = $(x+4)^2 - 4^2 - 3$ =

y = $(x+4)^2 - 19$

2. The minimum value of $(x+4)^2$ has to be 0.

Therefore the minimum value of y has to be 0 -19 = -19.

3. This occurs when x + 4 = 0. i.e. x = -4

4. So, the co-ordinates of your minimum point are (-4, -19)

4.7 Questions

Now find the co-ordinates for the minimum point of the following curves, using your answers from 4.4 to help you.

I. y = $x^2 - 8x + 12 = 0$
II. y = $x^2 + 10x + 21 = 0$
III. y = $x^2 - 4x - 5 = 0$
IV. y = $x^2 + 4x - 3$
V. y = $x^2 - 3x - 2 = 0$
VI. y = $x^2 + 12x - 1 = 0$
VII. y = $x^2 + 6x - 4 = 0$
VIII. y = $x^2 + 12x + 34 = 0$
IX. y = $x^2 - 10x + 15 = 0$
X. y = $x^2 - x - 1.75 = 0$

Simultaneous Equations (C1 and C2)

1) Elimination

1.1 Introduction

You should already be very familiar with this method of solving linear simultaneous equations from GCSE. This will again be tested at C1 and you will need to solve simultaneous equations in all of the M1 topics.

1.2 The Technique

e.g.1: $5x + 7y = 19$ **1** (x 3)
$3x + 2y = 7$ **2** (x 5)

In this example, the easiest way to proceed is to multiply equation **1** by 3 and equation **2** by 3. That will give us $15x$ in both:

$15x + 21y = 57$ ***1a***
$15x + 10y = 35$ ***2a***

Subtracting 2a from 1a:

$11y = 22$
$y = 11$

Now substitute this value into equation **2:** $3x + 2(11) = 7$
$3x + 22 = 7$
$3x = -15$
$x = -5$

1.3 Questions

I. $2x + 5y = 24$
$4x + 3y = 20$

II. $5x + 2y = 13$
$2x + 6y = 26$

III. $3x + y = 11$
$9x + 2y = 28$

IV. $x + 2y = 17$

8x + 3y = 45

V. 3x + 2y = 19
X + 8y = 21

VI. 2x + 3y = 9
4x + y = 13

VII. 2x + 7y = 17
5x + 3y = -1

VIII. 5x + 3y = 23
2x + 4y = 12

2) Substitution

2.1 Introduction

Although this technique can be used to solve linear equations, it is mainly used to solve simultaneous equations when 1 equation is quadric.

2.2 The Technique

Let's look at a fairly straight forward example to start with.

e.g. 1

$x^2 + y^2 = 25$ **1**
y = x + 1 **2**

1 Make either y or x the subject using the **linear** equation. This is easy in this example as we already know that y = x + 1

2 Now substitute y into equation **1** giving us:

$$x^2 + (x+1)^2 = 25$$

3 Now expand, simplify and solve.

$$x^2 + x^2 + 2x + 1 = 25$$
$$2x^2 + 2x - 24 = 0$$
$$x^2 + x - 12 = 0$$

$(x+4)(x-3)=0$

x = -4 or x = 3

It is important to substitute **both** values of x into either equation (normally the linear equation is easier) to find the corresponding values of y.

4 y = -4 + 1 = -3

y = 3 + 1 = 4

5 It is also important to show which values of x and y belong together. So, write your answers as something like this:

X = 4, y = -3 and x = -3, y = 4

e.g. 2

$xy+x^2=6$ **1**

3x + y = 7 **2**

1 y = 7 – 3x **2a**

[Here we make y the subject as it requires the least amount of substitution into equation 1]

2 Now substitute y into equation **1** giving us:

$x(7-3x)+x^2=6$

3 Now expand, simplify and solve.

$7x-3x^2+x^2=6$

$2x^2-7x+6=0$

$(2x-3)(x-2)=0$

X = 3/2 or x = 2

4 y = 7 – 9/2 = 5/2 (Using 2a)

y = 7 – 6 = 1

5 X = 3/2, y = 5/2 and x = 2, y = 1

2.2 Questions

I. $y = x^2 - 2x$
$y = x + 4$

II. $y = 7x - 8$
$y = x^2 - x + 7$

III. $y = x + 1$
$x^2 + y^2 = 13$

IV. $5x - y = 8$
$y = x^2 - 3x + 7$

V. $5x - y = 8$
$y = x^2 - 3x + 7$

VI. $y = x - 2$
$x^2 + y^2 = 20$

VII. $y = 4x - 8$
$y^2 = 16x$

Surds and Indices (C1, C2 and M1)

1) Indices

1.1 Introduction

There are a few simple rules you need to know that will enable you to work effectively with indices:

- $a^n \times a^m = a^{n+m}$ i.e. $x^4 \times x^9 = x^{13}$
- $a^n \div a^m = a^{n=m}$ i.e. $x^7 \div x^3 = x^4$
- $\left(a^n\right)^m = a^{n\times m}$ i.e. $\left(x^5\right)^3 = x^{15}$
- $a^0 = 1$ i.e. $7^0 = 1$
- $a^{-m} = \dfrac{1}{a^m}$ i.e. $x^{-4} = \dfrac{1}{x^4}$

1.2 Simplifying expressions

Using these rules, algebraic expressions can be simplified.

e.g. 1 $4x^3y^5 \times 5x^2y = 20x^5y^6$ (multiply coefficients first, then letters)

e.g. 2 $(3x^3y^4)^3 = 27x^9y^{12}$ (cube coefficient first, then letters)

e.g. 3 $\dfrac{9x^7y^6}{12x^3y^8} = \dfrac{3x^4}{4y^2}$ (simplify fractions as normal)

e.g. 4 $\dfrac{12x^7y^3}{6x^7y^4} = \dfrac{2}{y}$ (note how the x's cancel)

1.3 Questions

I. $7x^5y^4 \times 5x^3y^2 =$

II. $8x^8y^2z^2 \times 4x^4y^9z =$

III. $2x^5y^3z^2 \times 3x^4z \times 4xy^3 =$

IV. $(4x^3y^5)^3 =$

V. $(10x^3y^4z)^2 =$

VI. $(2xy^4)^5 =$

VII. $\dfrac{12x^2y^3}{18x^5y^4} =$

VIII. $\dfrac{12x^8y^6z^3}{15x^{10}y^3} =$

IX. $\dfrac{12x^6y^4z^3}{4x^5y^4z^4} =$

X. $\dfrac{3x^2y^5}{15x^5y^4} =$

1.4 Evaluating powers and indices

Sometimes, we are required to evaluate numbers which have been raised to a power. In addition to the rules outlined in section 1.1, you need to know the definition of a fractional power.

- $a^{\frac{1}{n}} = \sqrt[n]{a}$

e.g. 1 $9^{\frac{1}{2}} = \sqrt{9} = 3$

e.g. 2 $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

- $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$

e.g. 3 $8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$

e.g. 4 $16^{-\frac{3}{2}} = \dfrac{1}{16^{\frac{3}{2}}} = \dfrac{1}{\left(\sqrt{16}\right)^3} = \dfrac{1}{64}$

1.5 Questions

Evaluate the following:

I. 10^3

II. 2^4

III. 5^{-3}

IV. 7^0

V. $(-5)^2$

VI. $(-3)^3$

VII. $(-1)^5$

VIII. 4^{-3}

IX. 3^{-4}

X. 0.5^{-2}

XI. $125^{\frac{1}{3}}$

XII. $49^{\frac{1}{2}}$

XIII. $16^{-\frac{1}{4}}$

XIV. $125^{\frac{2}{3}}$

XV. $4^{\frac{3}{2}}$

XVI. $16^{\frac{3}{2}}$

XVII. $8^{-\frac{5}{3}}$

XVIII. $81^{-\frac{4}{3}}$

XIX. $\left(\dfrac{9}{4}\right)^{\frac{3}{2}}$

XX. $\left(\dfrac{27}{125}\right)^{-\frac{2}{3}}$

2) Surds

2.1 Introduction

Surds are numbers left in 'square root form'. They are therefore irrational numbers. The reason we leave them as surds is because in decimal form they would always have to be rounded and therefore inaccurate. Giving your answer in surd form means you are giving "the exact value". e.g $\sqrt{2}$ is much easier to write than 1.41421356…

2.2 Simplification of Surds

$\sqrt{9} \times \sqrt{4} = \sqrt{36} = 6$ [law of indices]
$\sqrt{7} \times \sqrt{11} = \sqrt{77}$
This rule is very useful when simplifying surds:

$\sqrt{32} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$ [always find the highest square number that divides into 32]

$\sqrt{75} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$

2.3 Multiplication of Surds

$3\sqrt{2} \times 5\sqrt{2} = 15\sqrt{4} = 30$ [just like algebra multiply the coefficients, then the surds]
$7\sqrt{3} \times 4\sqrt{3} = 28\sqrt{9} = 84$

$2\sqrt{3} \times 5\sqrt{2} = 10\sqrt{6}$
$3\sqrt{7} \times 5\sqrt{2} = 15\sqrt{14}$

$(3 + 2\sqrt{3})(2 - 4\sqrt{3})$ [The brackets are expanded as usual]
$= 6 - 12\sqrt{3} + 4\sqrt{3} - 12\sqrt{9}$
$= 6 - 8\sqrt{2} - 36$ [collect like terms and simplify]

$= -30 - 8\sqrt{2}$

2.4 Addition and Subtraction of Surds

Adding and subtracting surds are straightforward – like algebra the surds (like terms) need to be the same.

$4\sqrt{7} - 2\sqrt{7} = 2\sqrt{7}$.
$5\sqrt{2} + 8\sqrt{2} = 13\sqrt{2}$

However, sometimes you may have to simplify your surds first.

$\sqrt{12} + \sqrt{27}$

$= \sqrt{4}\,\sqrt{3} + \sqrt{9}\sqrt{3}$

$= 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$

2.5 Questions

Expand / simplify the following surds / expressions:

I. $\sqrt{27}$
II. $\sqrt{48}$
III. $\sqrt{72}$
IV. $\sqrt{200}$
V. $\sqrt{128}$
VI. $\sqrt{32} - 2\sqrt{2}$
VII. $\sqrt{75} + 4\sqrt{3}$
VIII. $3\sqrt{7} + \sqrt{28}$
IX. $6\sqrt{5} + \sqrt{125}$
X. $\sqrt{72} + 5\sqrt{2}$
XI. $(3+\sqrt{2})(5+\sqrt{2})$
XII. $(4+\sqrt{3})(5+2\sqrt{3})$
XIII. $(2-4\sqrt{3})(4+2\sqrt{3})$

XIV. $(5-4\sqrt{3})(5+4\sqrt{3})$
XV. $(7-2\sqrt{7})(7+2\sqrt{7})$

2.6 Rationalising the Denominator

If we have an algebraic fraction with a surd in the denominator, generally we want to get rid of this. By multiplying the top and bottom of the fraction by a particular expression, we can "rationalize" the denominator. This is because surds, by definition, are irrational numbers and so you are changing the denominator from an irrational to a rational number.

To rationalize a simple fraction with the surd as the denominator, multiply top and bottom of the fraction by the surd and simplify.

$$\frac{1}{\sqrt{2}}\ \frac{\times\sqrt{2}}{\times\sqrt{2}}=\frac{\sqrt{2}}{2}$$

$$\frac{9}{2\sqrt{3}}\ \frac{\times\sqrt{3}}{\times\sqrt{3}}=\frac{9\sqrt{3}}{6}=\frac{3\sqrt{3}}{2}$$

To rationalize a fraction with an expression such as 5 - √3, as the denominator, multiply top and bottom by the same expression but with the opposite sign, i.e. 5 + √3. This will result in the surd disappearing (difference of 2 squares):

(5 - √3) (5 - √3) = 25 – 3 = 22

Then simplify the fraction as before.

$$\frac{4}{5-\sqrt{3}}\ \frac{\times(5+\sqrt{3})}{\times(5+\sqrt{3})}=\frac{4(5+\sqrt{3})}{25-3}=\frac{4(5+\sqrt{3})}{22}=\frac{2(5+\sqrt{3})}{11}$$

$$\frac{4}{2+3\sqrt{2}}\ \frac{\times(2-3\sqrt{2})}{\times(2-3\sqrt{2})}=\frac{4(2-3\sqrt{2})}{4-18}=\frac{4(2-3\sqrt{2})}{-14}=-\frac{2(2-3\sqrt{2})}{7}$$

2.7 Questions

Rationalise the following:

I. $\frac{8}{\sqrt{2}}$

II. $\frac{10}{\sqrt{5}}$

III. $\frac{15}{\sqrt{3}}$

IV. $\frac{10}{7\sqrt{2}}$

V. $\frac{9}{2\sqrt{3}}$

VI. $\frac{10}{3\sqrt{5}}$

VII. $\frac{21}{4\sqrt{7}}$

VIII. $\frac{3}{\sqrt{2}+1}$

IX. $\frac{8}{\sqrt{5}-1}$

X. $\frac{9}{\sqrt{7}-2}$

XI. $\frac{6}{3\sqrt{2}+5}$

XII. $\frac{4}{2\sqrt{2}-1}$

XIII. $\frac{3}{4\sqrt{5}+3}$

XIV. $\frac{3}{5-3\sqrt{7}}$

XV. $\frac{3+2\sqrt{3}}{5-2\sqrt{3}}$

Graph Transformations (C1, C2 and C3)

1.1 Introduction

The transformation of graphs is quite a common topic throughout AS and A2 maths. The transformations you learn in C1 will be extremely useful for the transformations of trigonometric, exponential and logarithmic graphs you will come across in C2 and C3.

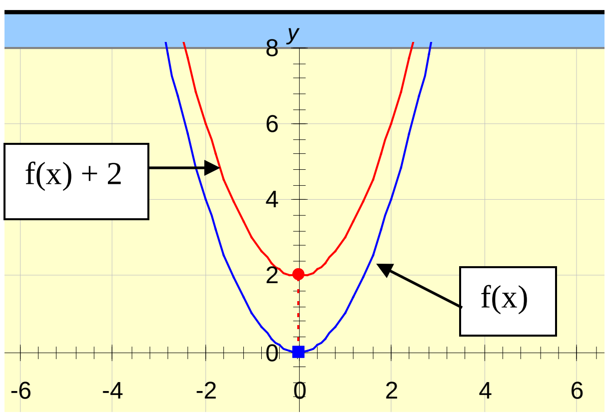
If you covered the entire "Higher" syllabus later year, you should already be familiar with the basic operations.

1.2 The 4 Transformations

1 $f(x)+a$

This is the easiest transformation. For a given graph of f(x), the transformation " $f(x)+a$ " simply **translates** the graph by "a" units in the positive y direction.

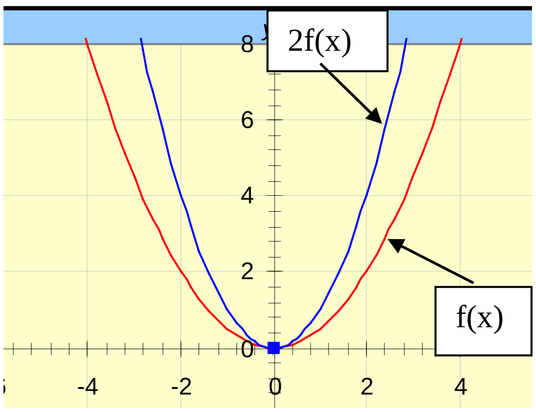
e.g. 1 - in this example f(x) +2 shows the original function has moved up by 2 units.



2 $af(x)$

For a given graph of f(x), the transformation " $bf(x)$ " means the graph is **stretched** by a factor of "a" in the y direction. In practice, this means all of the Y co-ordinates are multiplied by a factor of "a".

e.g. 2 - in this example 2f(x) shows the original function stretched by a factor of 2. For a given x co-ordinate, the y co-ordinate has been multiplied by 2.



3 $f(x+a)$

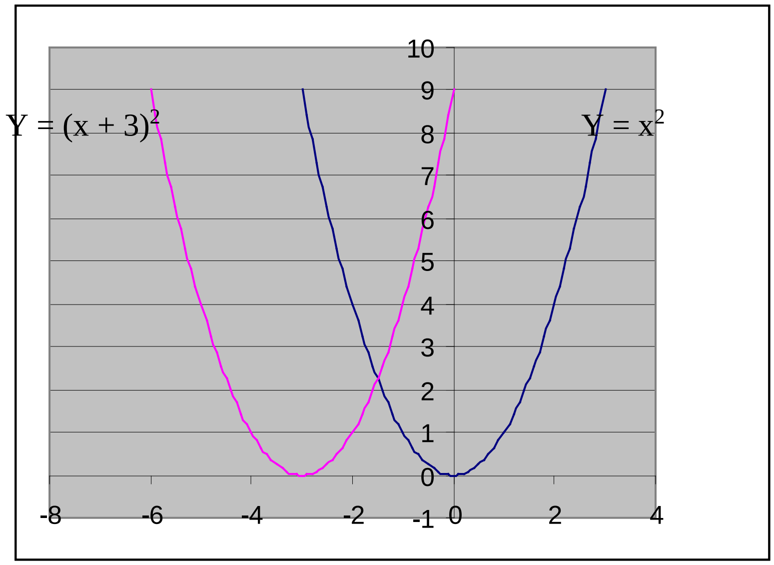
For a given graph of f(x), the transformation " $f(x+a)$ " means the graph is **translated** the graph by "a" units in the **negative** y direction. This seems instinctively wrong, as you'd think that $f(x+a)$ would move the graph to the right (positive x direction).

However, if you think about a graph of $y = x^2$ and $y = (x + 3)^2$

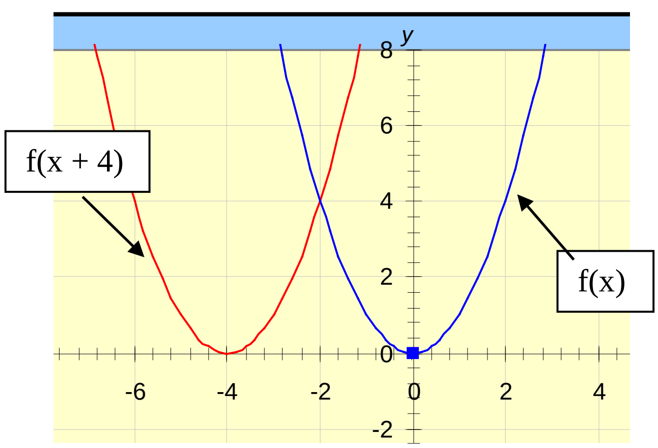
The graph of $y = x^2$ touches the x-axis when y = 0. So $x^2 = 0$, i.e. x = 0

The graph of $y = (x + 3)^2$ touches the x axis when $(x + 3)^2 = 0$, i.e. x + 3 = 0, so **x = -3**

Importantly, the new minimum point has moved to (-3, 0) and NOT (3, 0) as you might have thought.



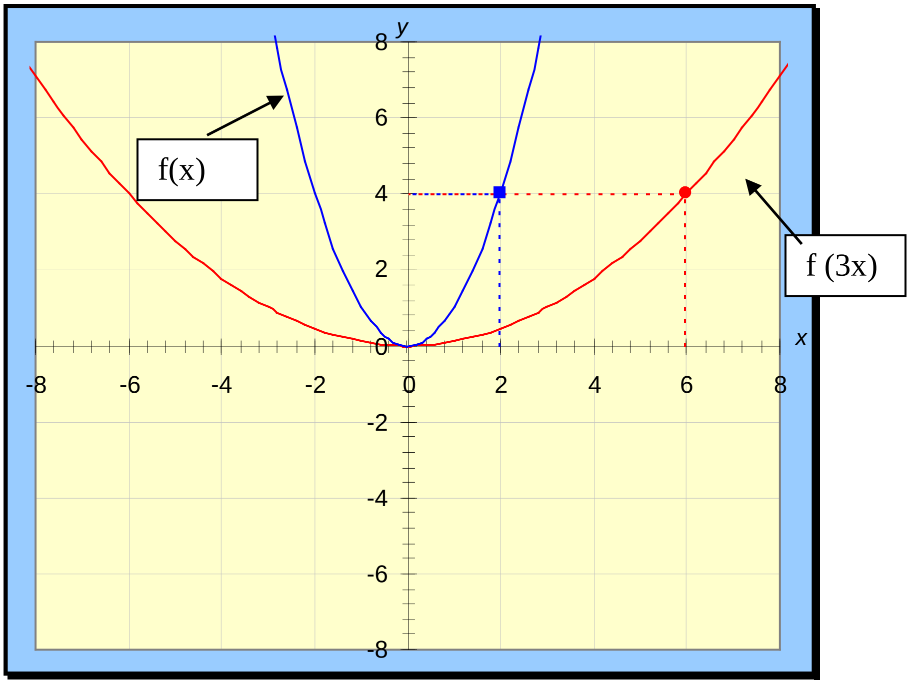
e.g. 3 - in this example f(x + 4) shows the original function translated by 4 units in the negative x direction.



4 $f(ax)$

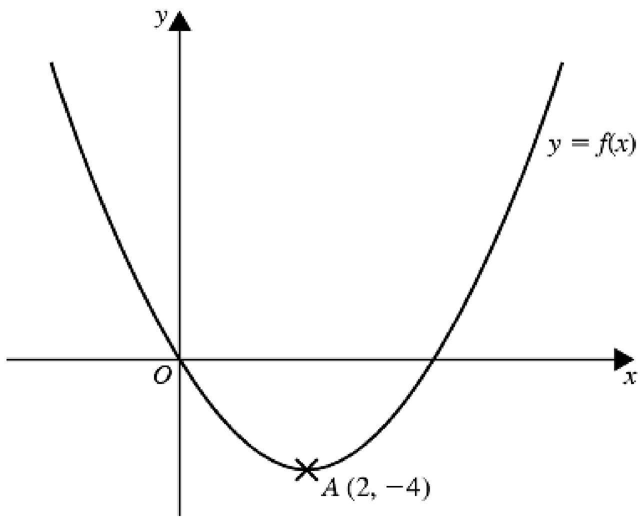
For a given graph of f(ax), the transformation " $f(ax)$ " means the graph is **stretched** by a factor of " $1/a$ " in the x direction. In practice, this means all of the x co-ordinates are multiplied by a factor of " $1/a$ ". An alternative way of thinking about this, is to **divide** all of your x-co-ordinates by *a*.

e.g. 4 - in this example f(3x) shows the original function stretched by a factor of $\frac{1}{3}$. For a given y co-ordinate, every x co-ordinate has been multiplied a factor of $\frac{1}{3}$ (or divided by 3).



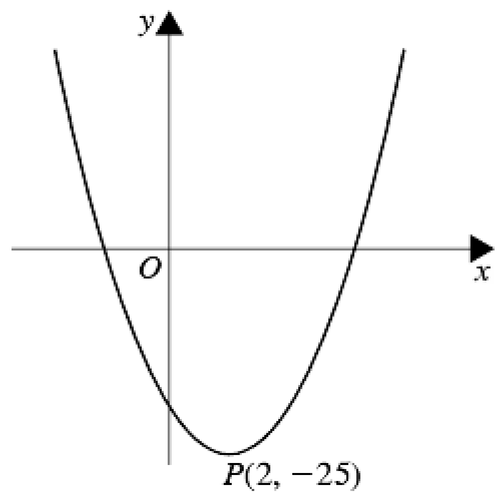
1.3 Questions

This is a sketch of the curve with equation $y = f(x)$.
It passes through the origin O.



The only vertex of the curve is at A (2, −4)
Write down the coordinates of the vertex of the curve with equation

I. $y = f(x - 3)$,
II. $y = f(x) - 5$
III. $y = f(2x)$
IV. $y = f(0.5x)$.
V. $y = f(x + 5)$.
VI. $y = 3f(x)$.
VII. $y = f(x) + 5$.
VIII. $y = 0.5f(x)$.
IX. $y = f(x) + 4$
X. $y = f(x + 3)$.



This is a sketch of the curve with equation $y = f(x)$.

The only vertex of the curve is $P(2, -25)$.

(a) Write down the coordinates of the vertex for curves with each of the following equations:

I. $y = f(x) + 4$
II. $y = f(x - 2)$
III. $y = 3f(x)$
IV. $y = f(2x)$
V. $y = f(x + 5)$
VI. $y = f(x) - 3$
VII. $y = f(0.5x)$
VIII. $y = f(x) - 8$
IX. $y = f(5x)$
X. $y = 0.5f(x)$